Exact calculations of gain and noise temperature vs. signal input frequency can then be made using (40), (41), (16), (17), and (26).

B. Approximate Analysis

For the approximate analysis, the assumption is made that the tuning range of signal input frequency is roughly equal to the loaded signal circuit 3-dB bandwidth. This bandwidth is in turn calculated approximately as follows. The complete signal circuit loop impedance in Fig. 11 is

$$Z_{\text{loop}} = Z_1 + R_s = R_1 + R_s + jX_1. \tag{42}$$

The fractional loaded signal circuit 3-dB bandwidth is then

$$\frac{(\Delta f_1)_{3\,\text{dB}}}{f_{10}} = \frac{1}{Q_1} \approx \frac{2(R_{10} + R_s)}{\omega_{10}\left(\dfrac{dX_1}{d\omega_1}\right)_{\omega_1=\omega_{10}}}. \tag{43}$$

From (41) and (6), (43) becomes

$$\frac{(\Delta f_1)_{3\,\text{dB}}}{f_{10}} \approx \frac{f_{10}(S_1+1)}{f_c}\left[\frac{\left(1+\dfrac{C_p}{C_0{}^s}\right)^2 + D_0{}^2\left(\dfrac{C_p}{C_0{}^s}\right)^2}{\left(1+\dfrac{C_p}{C_0{}^s}\right)^3 - D_0{}^4\left(\dfrac{C_p}{C_0{}^s}\right)^3}\right]. \tag{44}$$

A further approximation can be made if $\omega_{10}C_pR_{10} \ll 1 + (C_p/C_0{}^s)$ and $D_0 < 1$, namely

$$\frac{(\Delta f_1)_{3\,\text{dB}}}{f_{10}} \approx \frac{f_{10}(S_1 + 1)}{f_c\left(1 + \dfrac{C_p}{C_0{}^s}\right)}. \tag{45}$$

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Improved Intermodulation Rejection in Mixers

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***Abstract*—Intermodulation is one of the most pernicious forms of spurious response in superheterodyne receivers. This type of interference cannot be completely eliminated by narrowband filters. Improvements in receiver performance can be made only by improving and making more effective use of the mixing element.**

Intermodulation results from terms of higher order than two in the *v-i* power series expansion about the dc operating point of the mixing element. The magnitude of these higher order terms must be reduced in order to improve intermodulation rejection. In the work described in this paper, it has been observed that such a reduction in higher order terms can be obtained by proper design of the mixing element and by a proper choice of dc operating point.

More than 80 dB intermodulation rejection was obtained with a single ended hot carrier diode mixer. Best performance is obtained by operating two hot carrier diodes at different operating points in a balanced mixer arrangement. Intermodulation ratios greater than 100 dB have been measured for this operating mode. Optimization of performance for IM rejection has little effect on sensitivity or rejection of other spurious responses.

A new mixing element, the space-charge-limited resistor (SCLR), was designed to minimize the higher order terms in the current function. A balanced mixer provided the best performance with these mixing elements also. Intermodulation from +5 dBm inputs can be rejected below the mixer noise level. Early models of this device are not as sensitive as hot carrier mixers, but improvement appears to be possible.

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INTRODUCTION

THE INCREASING spectrum density of electromagnetic radiation has stimulated an interest in receivers which do not respond to signals which are not separated from the local oscillator frequency by the intermediate frequency. Such spurious responses result from combinations of harmonics of incoming signals with harmonics of the local oscillator. The general expression for two spurious input frequencies, f_1 and f_2, is

$$mf_2 - nf_1 = pf_{\text{LO}} \pm f_{\text{IF}}.$$

Narrowband receivers can be protected from many of these responses by preselector filters. For example, a common interference problem involves spurious inputs separated from the local oscillator frequency by submultiples of the intermediate frequency. That is, $n=0$ and $m=p$. The strongest response of this type is separated from the tuned frequency of the receiver by half the intermediate frequency and can be rejected by a properly designed filter. Other responses may be closer to the tuned frequency, but the amplitude decreases[1] so that the filter design remains feasible.

The intermodulation response, however, cannot be eliminated in this manner. This response is the result of the second harmonic of one input signal mixing with the fundamental of another input signal and the local oscillator. That is, $m=2$ and $n=p=1$, or $2f_2-f_1=f_0$ where f_0 is the tuned frequency of the receiver.

Since this relationship is satisfied for $f_1=f_0+2\delta$ and $f_2=f_0+\delta$ for any value of δ, the two spurious inputs may be arbitrarily close to the tuned frequency of the receiver. A typical mixer will respond to spurious inputs more than 50 dB above a desired signal. The techniques described below extend intermodulation rejection to 100 dB.

Opto-Electronic Mixing

The production of a difference frequency from two other frequencies in a mixer requires the multiplication of these frequencies. In a conventional mixer, this multiplication occurs in the cross products when the sum of the two frequencies is raised to the second or higher power. Intermodulation involves the sum of three frequencies, and the intermediate frequency results when this sum is raised to the fourth or higher power. In a photoconductor, mixing can result from a direct multiplication by modulating the conductance with one frequency and supplying the other frequency to the photoconductor circuit.[2] If the current is linearly related to the voltage across the photoconductor, the mixing will be free of spurious responses. An investigation of this device shows that there is a third-order term in the current function, which introduces intermodulation. The current is proportional to the number of carriers and the carrier velocity. The number of carriers is related to the illumination of the photoconductor so that the local oscillator frequency may be introduced by modulating this illumination. The carrier velocity is the product of carrier mobility and electric field strength. By modulating the electric field strength, the required multiplication may be obtained. Unfortunately, the mobility is a function of the square of electric field strength.[3] This relationship introduces a cubic term which combines with the conductivity factor to introduce a serious intermodulation component. The current expression is

$$I = G_0V(1 - bV^2)$$

where G_0 is proportional to the product of light intensity Φ and photoconductor lifetime τ, and b is related to the nonlinearity coefficient of the mobility. Intermodulation specification puts a lower limit on the product $\Phi\tau$, and sensitivity puts an upper limit on $\omega\tau$, where ω is the local oscillator frequency. The combination establishes a minimum requirement for the light intensity on the photoconductor. A sensitivity of -100 dBm and intermodulation rejection of 100 dB require $\Phi > 4.5 \times 10^{18}$ photons per second or one watt of light intensity for a 400 MHz local oscillator frequency. Even more light is required at higher frequencies. Other mixing devices appear to be more suitable for improved spurious suppression at this time.

Space-Charge-Limited Resistor

Since intermodulation requires the fourth or higher powers of voltage in a conventional mixer, the ideal mixing element would have no terms above the third power in the expansion of the current function. The space-charge-limited resistor[4] is basically a square-law device, although the variation of mobility with electric field introduces small third- and fourth-order terms. This device is made from n-type silicon with very high resistivity (10 000 ohm-cm) and good ohmic contacts.

The conversion efficiency of this device is limited by a linear term which behaves like series resistance in a conventional mixer diode. The effect of this term is minimized by using high resistivity and a thin layer of silicon.

A mixer using space-charge-limited resistors was built and did provide excellent rejection of intermodulation. Spurious input levels of one milliwatt were necessary to produce intermodulation above the noise level in a single ended mixer. In a balanced mixer, 10 milliwatt input signals were rejected below the noise level. The phenomenon of intermodulation rejection in a balanced mixer is discussed below.

Although the space-charge-limited resistor has excellent intermodulation rejection capabilities, it is not a satisfactory mixer at this time because its sensitivity is 30 dB worse than that of a hot carrier diode mixer. Theoretical studies[5] predict a noise temperature less than unity, but preliminary measurements of sensitivity and conversion efficiency indicate that most of the sensitivity loss is due to a high noise level.

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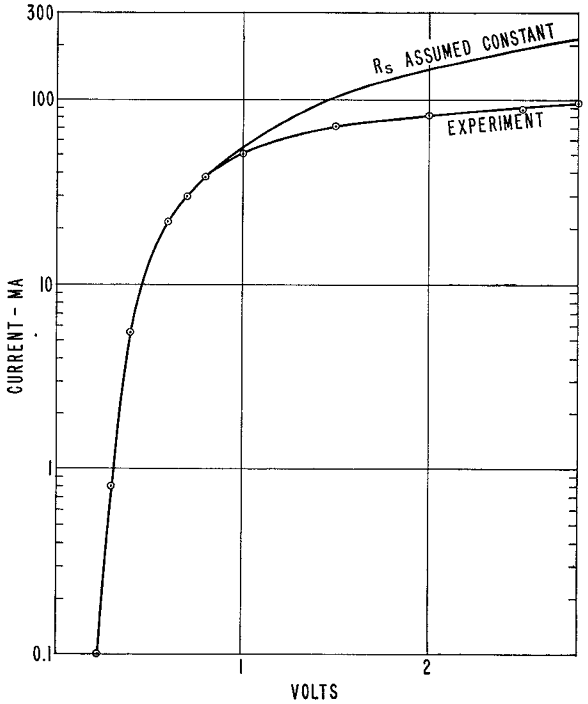

Fig. 1. Current response of a hot carrier diode.

Hot Carrier Diode

The current response of a typical hot carrier diode is shown in Fig. 1. The top curve is calculated from the expression

$$i = i_s[e^{\beta(v-iR_s)} - 1]$$

with the saturation current i_s, the series resistance R_s, and the constant β, chosen to fit the experimental data at low values of current. At higher current levels, the response drops below the theoretical curve. This is due to the fact that mobility is a decreasing function of electric field strength, so that the series resistance increases with current, instead of remaining constant as assumed in the theoretical curve. At still higher voltages, the current will tend to saturate at a value determined by the scatter limited velocity of majority carriers in the semiconductor.[6]

Since the current drops below the exponential value, the higher order terms in the Taylor expansion should be decreased with a resultant improvement in intermodulation rejection at high local oscillator drive levels. This is a further enhancement of performance by higher power in addition to that predicted for exponential diodes.[7] Figure 2 shows the intermodulation characteristics of a single ended hot carrier diode mixer. With 50 milliwatts of local oscillator drive and optimum bias setting, the intermodulation ratio is better than 80 dB. This is the ratio of spurious input power to desired input power for the same output level, 10 dB above the noise.

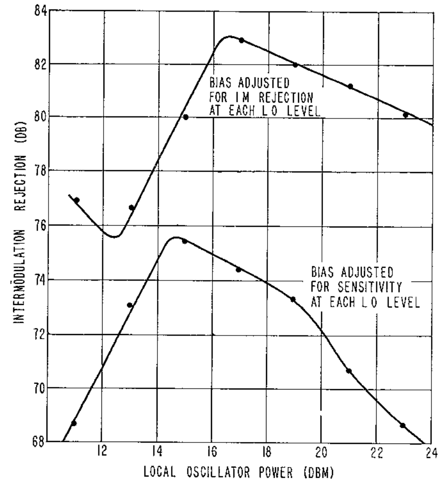

Fig. 2. Intermodulation response of singled ended mixer.

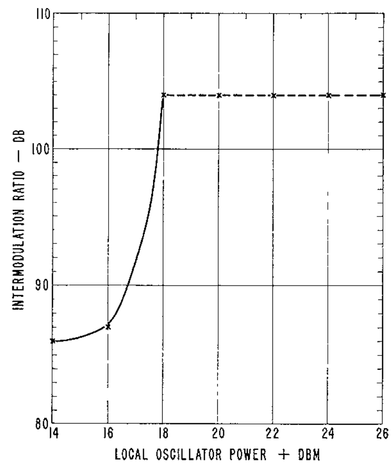

Fig. 3. Intermodulation rejection in a balanced mixer.

Better results are obtained when a balanced mixer is used. Figure 3 shows the intermodulation characteristics of a hot carrier balanced mixer with more than 100 dB of intermodulation rejection. Above +18 dBm of local oscillator power, the maximum input power of 5 milliwatts was used and the intermodulation was below the noise level. In the single ended mixer characteristic the slope of the intermodulation vs. local oscillator power curve is 2, while in the balanced mixer case the slope is about 20.

Analysis of Spurious Rejection in Balanced Mixers

The cancellation of intermodulation in a balanced mixer is an apparent contradiction of balanced mixer theory, which predicts summation of this response.[8] Cancellation indicates the presence of an additional phase reversal between the intermediate frequency outputs of the two diodes. If most of the intermodulation is produced by a particular higher order term in the

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Taylor expansion of the current function, the required phase change is equivalent to a change in sign of the coefficient in question. Assuming that the fourth- and sixth-order terms are responsible for intermodulation, Fig. 4 shows how the coefficients might vary with bias to produce a better intermodulation ratio in a balanced mixer than in a single ended mixer. Bias points V_1, V_2, and V_3 could produce IM reduction in single ended mixer operation, since they all minimize one of the even-order coefficients. In balanced mixer operation, selection of bias points V_1 and V_2 provides both *minimization* of k_4, and *cancellation* of the IM response due to k_6.

It now remains to be shown that some current function $f(v)$ can actually produce the type of behavior depicted in Fig. 4. The following simple analysis shows that in fact such a current function does exist. Given a function $f(v)$, the fourth- and sixth-order coefficients in the series expansion about a bias V are as follows:

$$k_4 = \frac{1}{4!}\left.\frac{d^4f}{dv^4}\right|_V = \frac{1}{4!}f^{(4)}(V) \quad (1)$$

$$k_6 = \frac{1}{6!}\left.\frac{d^6f}{dv^6}\right|_V = \frac{1}{6!}f^{(6)}(V). \quad (2)$$

The coefficients k_4 and k_6 are evidently related by the equation

$$k_6 = \frac{1}{30}\frac{d^2k_4}{dV^2}\cdot \quad (3)$$

Let us postulate that k_6 is a *linear* function of bias V. This is the simplest function for k_6 that can satisfy our requirements; we will find, as a result of the analysis to follow, that this choice does not explain the experimental results, and that a slightly more complicated form must be used. The approach leading to this result is interesting, however, and will be reproduced here. By (3), k_4 must be a cubic in V if k_6 is linear; and k_4 can be expressed as follows:

$$k_4 = (V - r_1)(V - r_2)(V - r_3) \quad (4)$$

where r_1, r_2, and r_3 will be determined by the requirement that k_6 have equal but opposite values at two of the roots of k_4, say r_1 and r_2. Differentiating (4) twice yields, after rearrangement:

$$k_6 = \frac{6}{30}\left(V - \frac{r_1 + r_2 + r_3}{3}\right). \quad (5)$$

We now require

$$k_6(r_1) = -k_6(r_2). \quad (6)$$

Combining (5) and (6), we can solve for r_3:

$$r_3 = \frac{r_1 + r_2}{2}\cdot \quad (7)$$

Equation (7) states that imposing condition (6) on k_6 forces the third root of k_4 to be the arithmetic mean of

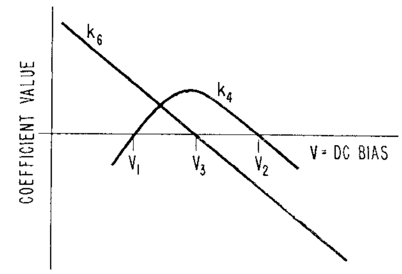

Fig. 4. Desired variation of coefficient values with dc bias. This qualitative behavior is necessary to explain IM reduction in single-ended mixer operation, and IM cancellation and reduction in balanced mixer operation.

the two roots at which we required k_6 to have equal and opposite values. Substitution of (7) into (5) gives

$$k_6 = \frac{6}{30}\left(V - \frac{r_1 + r_2}{2}\right) = \frac{6}{30}(V - r_3), \quad (8)$$

i.e., the single root of k_6 automatically turns out to be the same as one of the roots of k_4. If this were the case in an actual mixer element, one could bias the device at $V = r_3$ and simultaneously eliminate k_4 and k_6, the dominant terms in the production of the IM response under consideration, and so no improvement over single-ended operation would be expected.

The identical root for k_4 and k_6 is an unavoidable consequence of the choice of a linear function for k_6; a slightly more complicated function of V for k_6 leads to an explanation of the observed results. If we choose for k_6 a quadratic function of V, then k_4 must have the form

$$k_4 = (V - r_1)(V - r_2)(V - r_3)(V - r_4). \quad (9)$$

Differentiating twice, we obtain for k_6

$$k_6 = \frac{2}{30}[(V - r_1)(V - r_2) + (V - r_1)(V - r_3) + (V - r_1)(V - r_4) + (V - r_2)(V - r_3) + (V - r_2)(V - r_4) + (V - r_3)(V - r_4)]. \quad (10)$$

Requiring $k_6(r_1) = -k_6(r_2)$ yields the equation

$$2(r_1 - r_2)^2 + (r_1 - r_3)(r_1 - r_4) + (r_2 - r_3)(r_2 - r_4) = 0. \quad (11)$$

Thus, r_1, r_2, and r_3 may be chosen arbitrarily, and (11) determines r_4. For illustrative purposes we may choose

$$r_1 = 1, \quad r_2 = 2, \quad r_3 = 3.$$

Solving (11) yields $r_4 = \frac{2}{3}$, so the complete expression for k_4, including an arbitrary multiplying factor, is

$$k_4 = \frac{1}{96}(V - 1)(V - 2)(V - 3)(V - \tfrac{2}{3}). \quad (12)$$

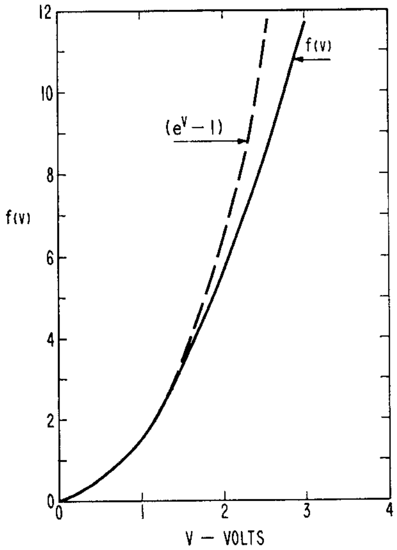

Fig. 5. The current function $f(v)$ which gives the coefficients shown in Fig. 6. Exponential current function is shown for comparison.

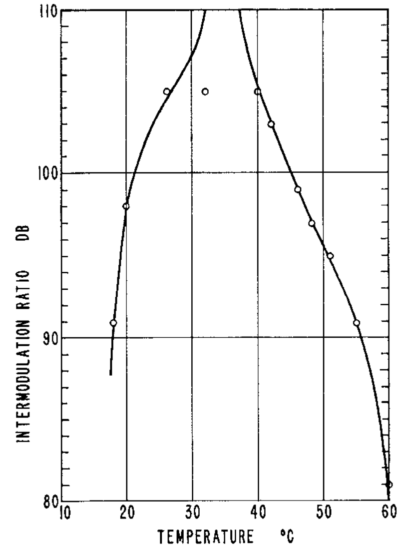

Fig. 7. Temperature dependence of spurious rejection by a hot carrier mixer.

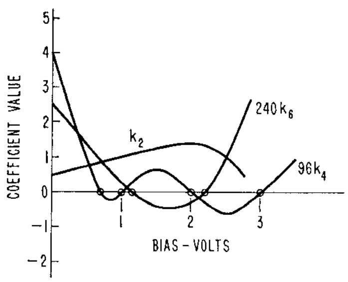

Fig. 6. Variation of k_2, k_4, and k_6, with bias voltage V for $V_1=1$, $V_2=2$, $V_3=3$, $V_4=\frac{2}{3}$.

Four successive integrations of k_4 with respect to V yield the original current function $f(V)$:

$$f(V) = 4!\iiiint k_4\, dV. \tag{13}$$

Substitution of (12) for k_4, and arbitrarily choosing all constants of integration to be unity, yields

$$(V) = V + \frac{1}{2!}V^2 + \frac{1}{3!}V^3 + \frac{1}{4!}V^4 - \frac{1}{36}V^5 + \frac{1}{96}V^6 - \frac{1}{504}V^7 + \frac{1}{(35)(48)(4)}V^8 \tag{14}$$

and

$$k_6 = \frac{1}{240}(V - 2.19)(V - 1.14). \tag{15}$$

The function $f(V)$ is plotted in Fig. 5, and k_4 and k_6 are plotted in Fig. 6. Examination of Fig. 5 discloses a form which is qualitatively similar to the hot carrier diode current function. An important feature is that the characteristic *departs from exponential* in the region of high bias; this condition is necessary for coefficient cancellation, and it is of interest to note that measured dc characteristics of hot carrier and point-contact diodes show this effect, as exemplified by Fig. 1. It is not difficult to see that pure exponential behavior will produce neither the desired cancellation nor nulling effect.

Referring to Fig. 6, we see that k_4 and k_6 indeed exhibit the desired behavior at $V=1$ and $V=2$; k_4 becomes zero at both points, and k_6 has exactly equal and opposite values. Since the behavior of the *desired* IF response must also be known as a function of bias, k_2 has been calculated and plotted. For a given LO drive and desired signal input power, IF response is mainly dependent on k_2; examination of Fig. 6 shows that k_2 does not change by more than a factor of 3 in the region $0<V<2.5$. Thus, the sensitivity of our fictitious mixer element would not be changed by more than a few dB by adjustment of the bias for maximum IM rejection.

Variation of ambient temperature has little effect on noise figure of a hot carrier diode mixer. A temperature range of 100°C produces less than 0.5 dB change of noise figure. Figure 7 shows the temperature sensitivity of the intermodulation characteristic. Rejection remains high for a 25°C range. Operation over wider temperature ranges may be possible by using thermistors in the bias circuit.

Conclusion

A technique has been described for improving the intermodulation rejection characteristics of mixers by more than 50 dB. Table I summarizes the results. The technique involves the operation of a balanced mixer with the two mixing elements at different bias points. The analysis involves control over the coefficients in the Taylor expansion of the current function. Experimental evidence is described for mixers using space-charge-limited resistors and hot carrier diodes. The analysis is applicable to other mixing devices and has been successfully applied to the insulated gate field effect transistor, for example.

TABLE I

	Intermodulation Rejection (dB)
Conventional Operation	50
New Technique Single Diode	80
New Technique Balanced Mixer	100

The measurements described were made in the 200 MHz region, but the technique is applicable at any frequency range where similar mixing devices are available.

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Accurate Phase-Length Measurements of Large Microwave Networks

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Abstract—The Stanford two-mile linear accelerator uses 240, single-input-port, four output-port, S-band, rectangular-waveguide networks to feed RF energy sixty to seventy feet from the klystrons above ground to ten-foot-long, disk-loaded, circular-waveguide, accelerator sections below ground. During installation it is necessary to permanently adjust the phase lengths of the four network branches to be within ±4.5 electrical degrees of the design lengths for RF wave and electron beam synchronization.

A modulated reflection phase-length comparison method is used, whereby a small signal is sent into each branch and reflected, in turn, by a diode switch, which is turned on and off at a 1 kHz rate. A null occurs in the amplitude modulation of the sum of a large reference signal and the small reflected signal, when the two signals are nearly in-phase quadrature. The reflectors are placed so that the network branches are properly adjusted when the nulls from all branches occur for the same setting of a variable phase shifter in the measurement line.

Small mismatches and multiple power divisions do not affect the accuracy of this method. Frequency, temperature, and air pressure are the main environmental conditions affecting the measurement and are discussed along with the design of the reflecting diode switch, which is mounted in a vacuum-sealed waveguide flange.

I. INTRODUCTION

OFTEN THE accurate measurement of electrical phase lengths is necessary in the design of microwave circuits. For numerous components and devices, a short length of transmission line is needed to act as a transformer to match two impedances. In other cases, where the load and the generator are matched already to the characteristic impedance of the transmission line, but the generator feeds several output ports, it may be important that the signals arrive at the respective ports with specific time delays or phase relationships. The system described below is designed to measure this latter type of network.

The method used to make an electrical phase-length measurement depends upon the type of network. Relative or comparison phase-length measurements generally are easier to make than absolute phase-length measurements, which may require detailed knowledge of the device's phase vs. frequency response. Thus, the phase length usually is determined by comparison with a similar, known or reference phase length in conjunction with an accurately calibrated, variable phase shifter.

The device to be measured may be inserted into the measurement circuit, so that the test signal either is transmitted through the device, or sent through the device and reflected back through it by a reflector on the output. A reflection method measures twice the length, and thus half-cycle phase length differences appear as full-cycle phase shifts. This phase ambiguity in large networks, with uncertain phase-frequency characteristics, is not resolvable simply by measuring the phase length at several frequencies. A transmission phase measurement method for a large network usually requires that a long phase-stable return path be provided from the output port of the network back to the phase measurement system. Often with both methods, effects due to small undesired CW reflections and unequal circuit attenuation can be minimized by modulating the signal. A more complete discussion of various phase-length measuring techniques is given by Lacy, and others [1]–[7].

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